TM-70-2014-5

TECHNICAL MEMORANDUM

LUNAR ORBIT DETERMINATION FOR APOLLO 12 USING POLAR

Bellcomm

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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Lunar Orbit Determination for Apollo 12 Using POLAR

TM- 70-2014-5

DATE- March 31, 1970

FILING CASE NO(S)- 310

AUTHOR(S)- A. J. Ferrari
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FILING SUBJECT(S) (ASSIGNED BY AUTHOR(S))- Orbit Determination

ABSTRACT

The Doppler errors of two pass solutions obtained from the POLAR version of the Osculating Lunar Elements Program were small (.8 fps peak-to-peak) and two pass predictions resulted in only slight growth in these errors. The presence of a high correlation in the solution matrix between the m_o and I_o parameters could be responsible for this error growth. The Doppler errors of two separate ten pass solutions were of the same systematic nature and magnitude as those of two pass solutions.

Comparisons made in local vertical coordinates between two pass local solutions and two pass predictions show out-of-plane dispersions to be large (W_{max} = ±13,000 ft) and down-range dispersions to have a negative bias (δV_{max} = -2000 ft). Orbital element comparisons show significant differences existed among both the inclination and anomaly parameters.

A comparison between the orbital elements of a ten pass solution and those of a series of two pass solutions shows excellent agreement with the exception of the anomaly and inclination terms. Doppler insensitivity to the orbital inclination precludes the addition of the necessary linear inclination term required for planar consistency.

BR-145A (8-68)

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1.0 INTRODUCTION

The orbit determination and prediction capabilities of the POLAR version of the Osculating Lunar Elements Program* are investigated using Command Service Module (CSM) Doppler tracking data acquired during the Lunar Parking Orbit of Apollo 12. The navigation qualities of these orbit determinations are evaluated by comparing both the orbital elements and the rectangular states (rotated into local vertical coordinates) of the predicted and local solutions. This memorandum presents an analysis of the results obtained.

2.0 DATA ANALYSIS

POLAR orbit determinations were performed by fitting two consecutive passes of tracking data. The solution associated with each orbit determination was then used to predict the Doppler observable for the next two consecutive data passes. The two pass solutions and predictions were obtained using Apollo 12 data from lunar front side passes 3 through 12. Two additional orbit determinations were performed using ten passes of tracking data in each case (passes 3-12 and passes 20-29).

2.1 Two Pass Solutions

For the case of two pass solutions, the CSM trajectory is represented by the following nine solution parameters:

$$\{e_{co}, e_{c1}, e_{so}, e_{s1}, I_o, \Omega_o, \Omega_1, m_o, m_1\}$$

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*Bullock, M. V. and Ferrari, A. J., "An Analysis of Apollo 10 Tracking Data Utilizing the POLAR Version of the Osculating Lunar Elements Program" TM-69-2014-9, September 30, 1969, Case 310.

The linear inclination term, I_1, was excluded from the solution set because it became highly correlated (ρ=.99) with the orbital motion parameter, m_1.

The residuals associated with each two pass solution are systematic and possess a peak-to-peak amplitude of about 0.8 fps. These residuals are very similar in appearance to short-period gravitational variations not modeled in POLAR. The fit and prediction quality of the solutions is shown in the representative residual plots presented in Figure 1. The residual growth, though small, experienced for some solutions is probably due to the extremely high correlation (ρ=.999) existing between the constant parts of the anomaly (m_o) and the inclination (I_o) parameters. This correlation reflects the insensitivity of the observable to the orbital inclination for the tracking coverage and trajectory of Apollo 12.

2.2 Ten Pass Solutions

For the case of orbit determinations derived from longer data intervals, the POLAR parameter set was expanded to allow for long-period variations in the elements. The parameter set used for both of the ten pass orbit determinations performed consisted of the following twelve terms:

$$\{e_{so}, e_{s1}, e_{s2}, e_{co}, e_{c1}, e_{c2}, \Omega_o, \Omega_1, I_o, m_o, m_1, m_2\}$$

Again the linear inclination parameter was excluded from the solution set since it was highly correlated with the orbital motion term, m_1. The Doppler residuals resulting from the ten pass orbit determinations (Figure 2) are 0.9 fps peak-to-peak and again have the systematic short-period variations characteristic of the two pass solutions.

3.0 SOLUTION COMPARISONS

Solution consistency was studied by forming differences between predicted and local two pass solutions in both the local vertical coordinate system (U-radial, V-downrange, W-out-of-plane) and in the orbital state. A comparison was also made between the orbital elements of the ten pass solution (Revs. 3-12) and those of the two pass solutions taken over that data.

3.1 Two Pass Solution Comparisons

Local vertical deviations in position and velocity for POLAR two pass solutions are shown in Figures 3a and 3b respectively. The table lists the comparisons and an associated numerical designator

No.	Solution		* Prediction
1.	5-6	vs.	$\hat{5}$-$\hat{6}$
2.	6-7	vs.	$\hat{6}$-$\hat{7}$
3.	7-8	vs.	$\hat{7}$-$\hat{8}$
4.	8-9	vs.	$\hat{8}$-$\hat{9}$
5.	9-10	vs.	$\hat{9}$-$\hat{10}$

*Hat (^) indicates extrapolation. For example, $\hat{5}$-$\hat{6}$ denotes the solution from passes 3-4 propagated forward to passes 5-6.

The smallest deviations obtained were in the radial (U) component (δU_{max} = +450 ft to -300 ft). The magnitude of these deviations reflects a consistency in the semi-major axes and eccentricities of the POLAR two pass solutions. All of the solutions compared have a negative bias associated with the down-range (V) dispersions. These errors vary in magnitude from δV_{min} = -200 ft to δV_{max} = -2400 ft. The presence of large down-range biases among the POLAR solutions is a manifestation of dispersions among the m_o parameters. These dispersions are a result of linear combinations existing between the m_o and I_o terms. The out-of-plane (W) deviations are very sinusoidal in nature and exhibit the largest difference in the POLAR solutions (δW_{max} = ±13,000 ft). These large dispersions are a result of differences among the planar parameters.

The orbital elements from each two pass solution were extrapolated forward to the beginning of the next pass and compared with the local two pass solution at that epoch. The results of these comparisons are presented below using the same numerical designators previously introduced:

Semi-Major Axis: (a)

$\delta a_1 = 6$ ft.

$\delta a_2 = -13$ ft.

$\delta a_3 = 11$ ft.

$\delta a_4 = 9$ ft.

$\delta a_5 = 36$ ft.

Eccentricity: (e)

$\delta e_1 = .3313 \times 10^{-4}$

$\delta e_2 = .1480 \times 10^{-4}$

$\delta e_3 = .07618 \times 10^{-4}$

$\delta e_4 = .16513 \times 10^{-4}$

$\delta e_5 = .1339 \times 10^{-4}$

Inclination: (I)

$\delta I_1 = 0.1370°$

$\delta I_2 = 0.0945°$

$\delta I_3 = 0.0374°$

$\delta I_4 = 0.0500°$

$\delta I_5 = 0.1000°$

Longitude of Ascending Node: (Ω)

$$\delta\Omega_1 = 0.0005710°$$

$$\delta\Omega_2 = 0.006550°$$

$$\delta\Omega_3 = 0.005196°$$

$$\delta\Omega_4 = 0.002037°$$

$$\delta\Omega_5 = 0.017195°$$

Anomaly Angle: $(m=M+\omega)$

$$\delta m_1 = 0.01513°$$

$$\delta m_2 = 0.01335°$$

$$\delta m_3 = 0.005098°$$

$$\delta m_4 = 0.006775°$$

$$\delta m_5 = 0.016291°$$

Analysis of these orbital element dispersions shows that the large out-of-plane inconsistencies are a result of differences in the values of the inclination parameter. The down-range biases mentioned earlier are directly related to the rather large anomaly angle $(M+\omega)$ dispersions obtained.

3.2 Ten Pass Solution Comparison

Comparisons between the time-varying orbital elements derived from a ten pass solution and those of the corresponding local two pass solutions show that these elements all have very good agreement with the exception of the inclination and anomaly parameters (see Figures 4a-f). Lack of sensitivity of the Doppler data to orbital inclination has resulted in a degradation of estimates for these elements.

4.0 CONCLUSIONS

Two pass predictions obtained from POLAR two pass solutions resulted in little or no growth in the Doppler errors. High correlation did exist in the solution matrix between the constant part of the anomaly and inclination parameters. Solutions obtained from processing ten passes of data possess residual characteristics and amplitudes similar to those of two pass solutions.

Local vertical comparisons among two pass local solutions and extrapolated solutions show out-of-plane dispersions to be large and down-range dispersions to possess a negative bias. Analysis of the orbital elements of these solutions shows that dispersions among the inclination parameter, I_o, and the constant part of the anomaly, m_o, do exist. These inclination and anomaly errors are responsible for the large out-of-plane errors and the down-range biases obtained. The high correlation in the solution matrix is an indication that a linear combination exists between these solution parameters.

Comparisons made between the orbital elements of a ten pass solution and those of a series of two pass solutions show excellent agreement with the exception of the anomaly and inclination elements. Doppler insensitivity precludes the addition of the necessary linear inclination term for planar consistency.

A. J. Ferrari

A. J. Ferrari

M. V. Bullock

2014-$\frac{\text{AJF}}{\text{MVB}}$-ksc M. V. Bullock

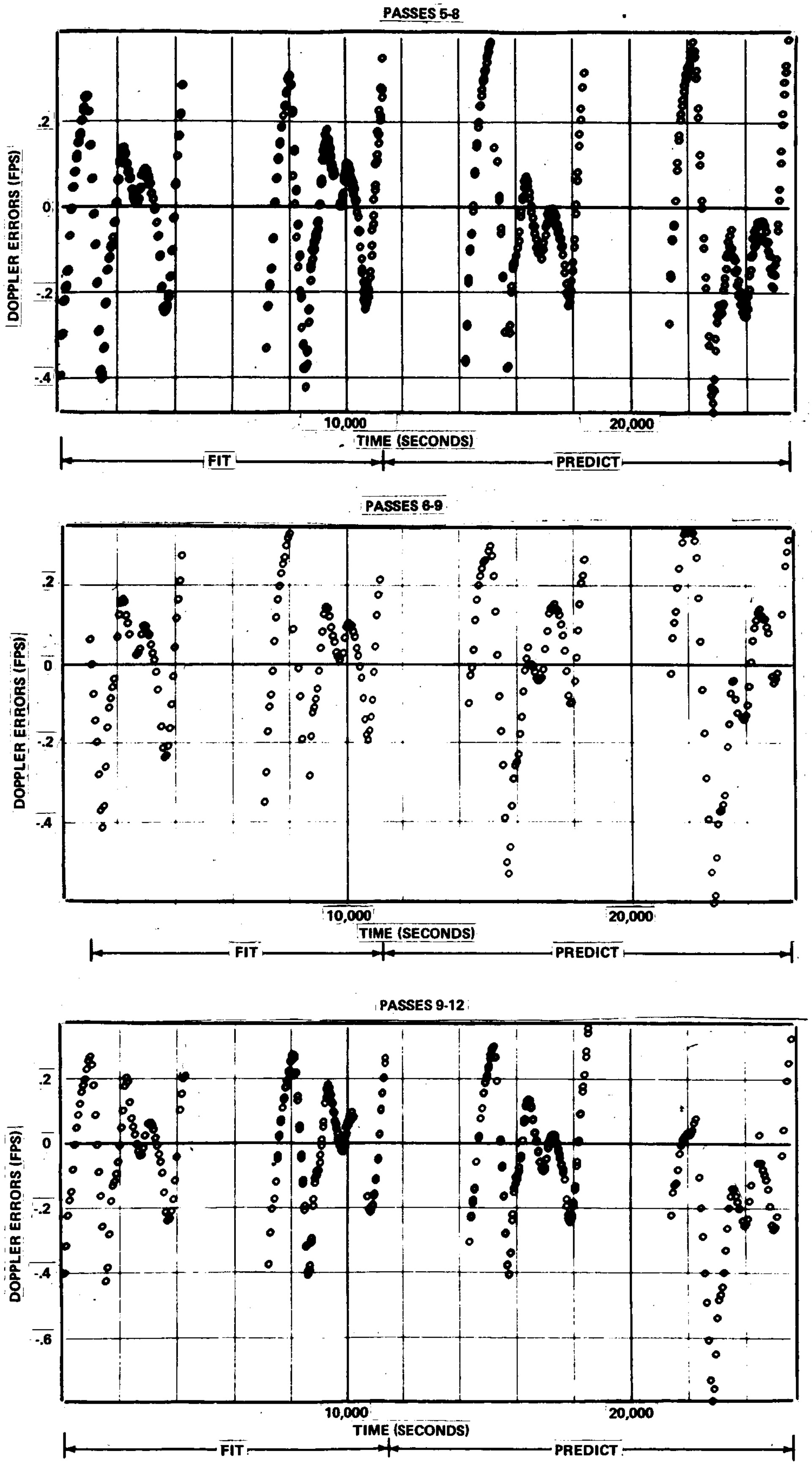


FIGURE 1 - DOPPLER RESIDUALS FROM TWO PASS SOLUTIONS AND PREDICTIONS

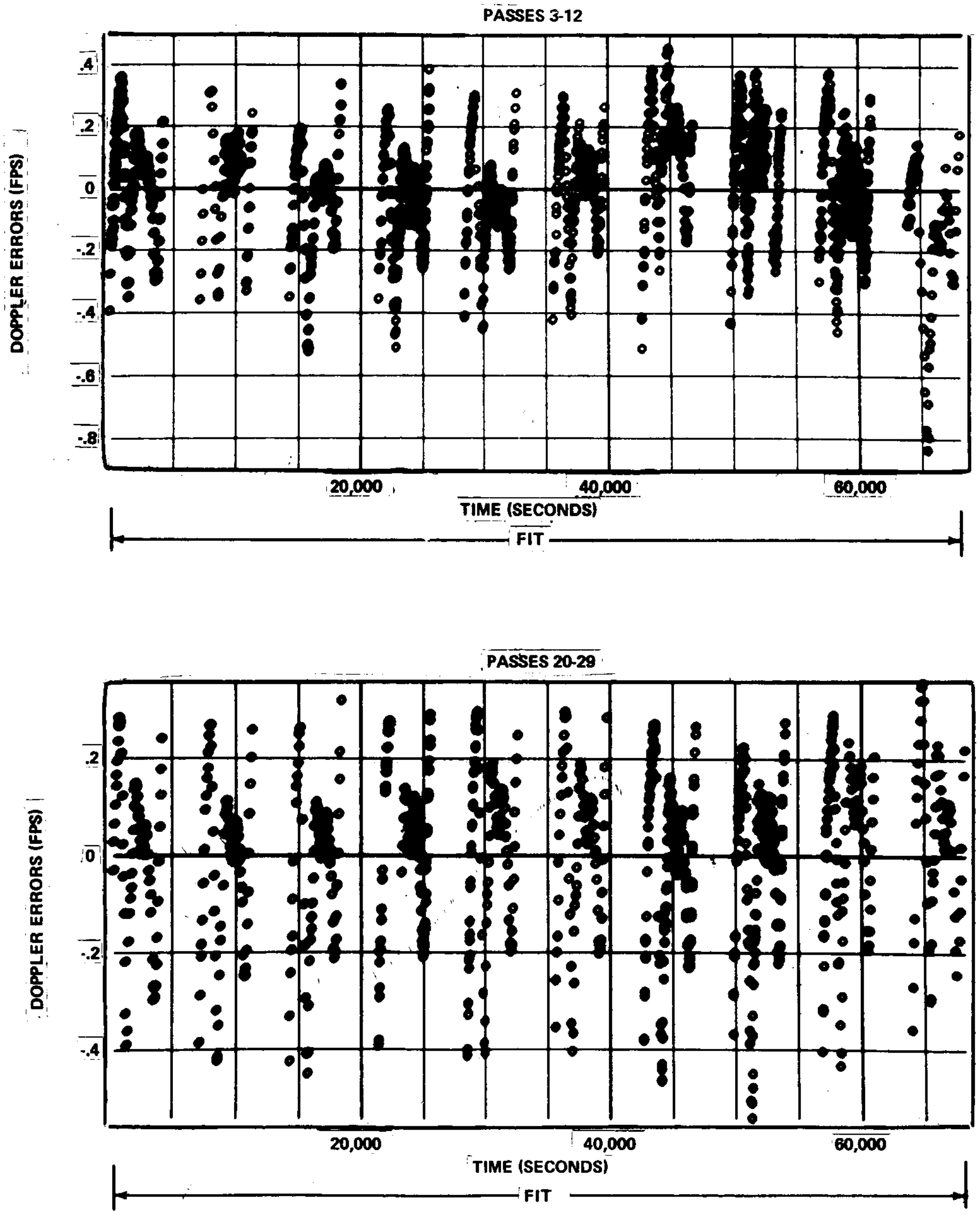


FIGURE 2 - DOPPLER RESIDUALS FROM MULTI-PASS SOLUTIONS

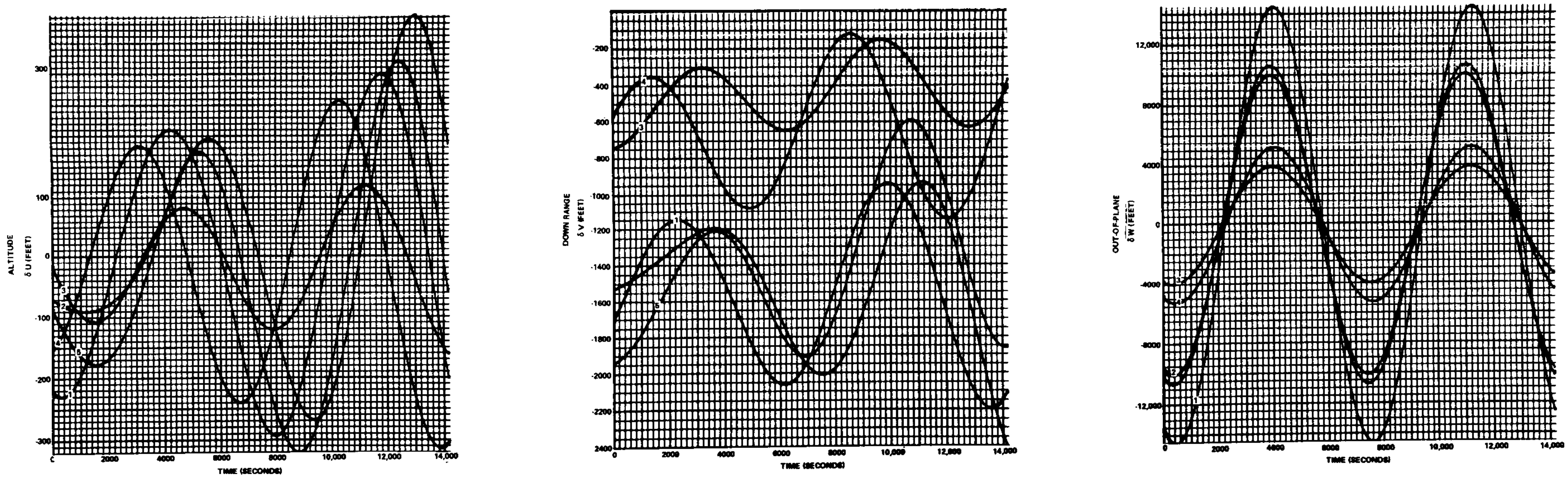


FIGURE 3a - LOCAL VERTICAL COMPARISONS (POSITION)

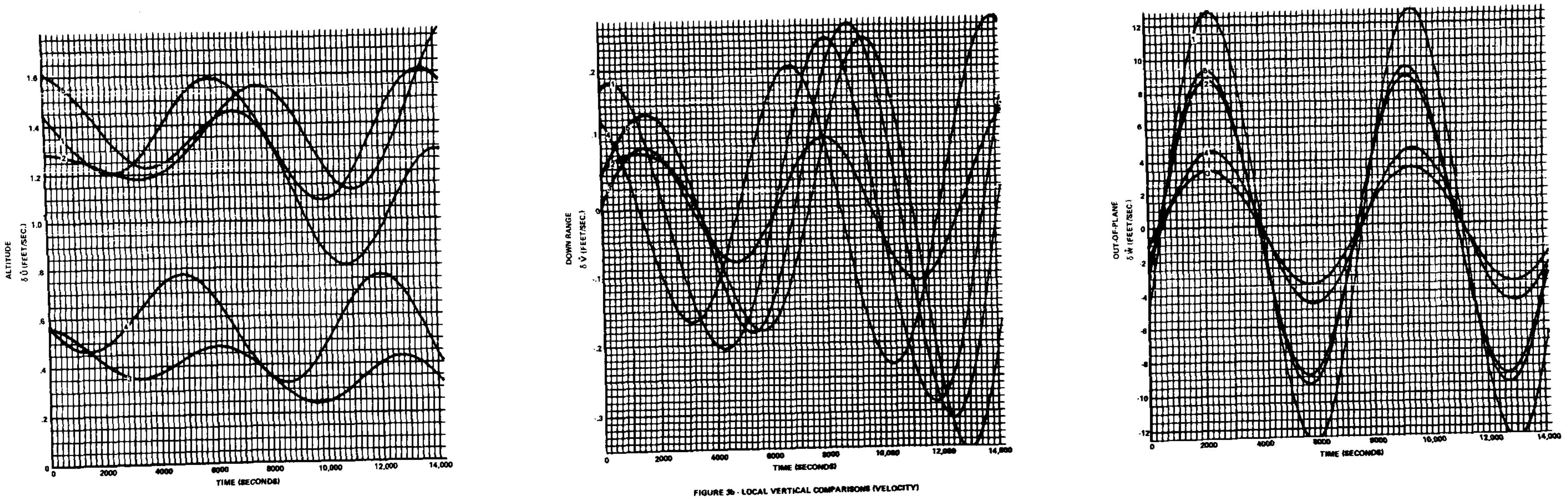


FIGURE 3b - LOCAL VERTICAL COMPARISONS (VELOCITY)

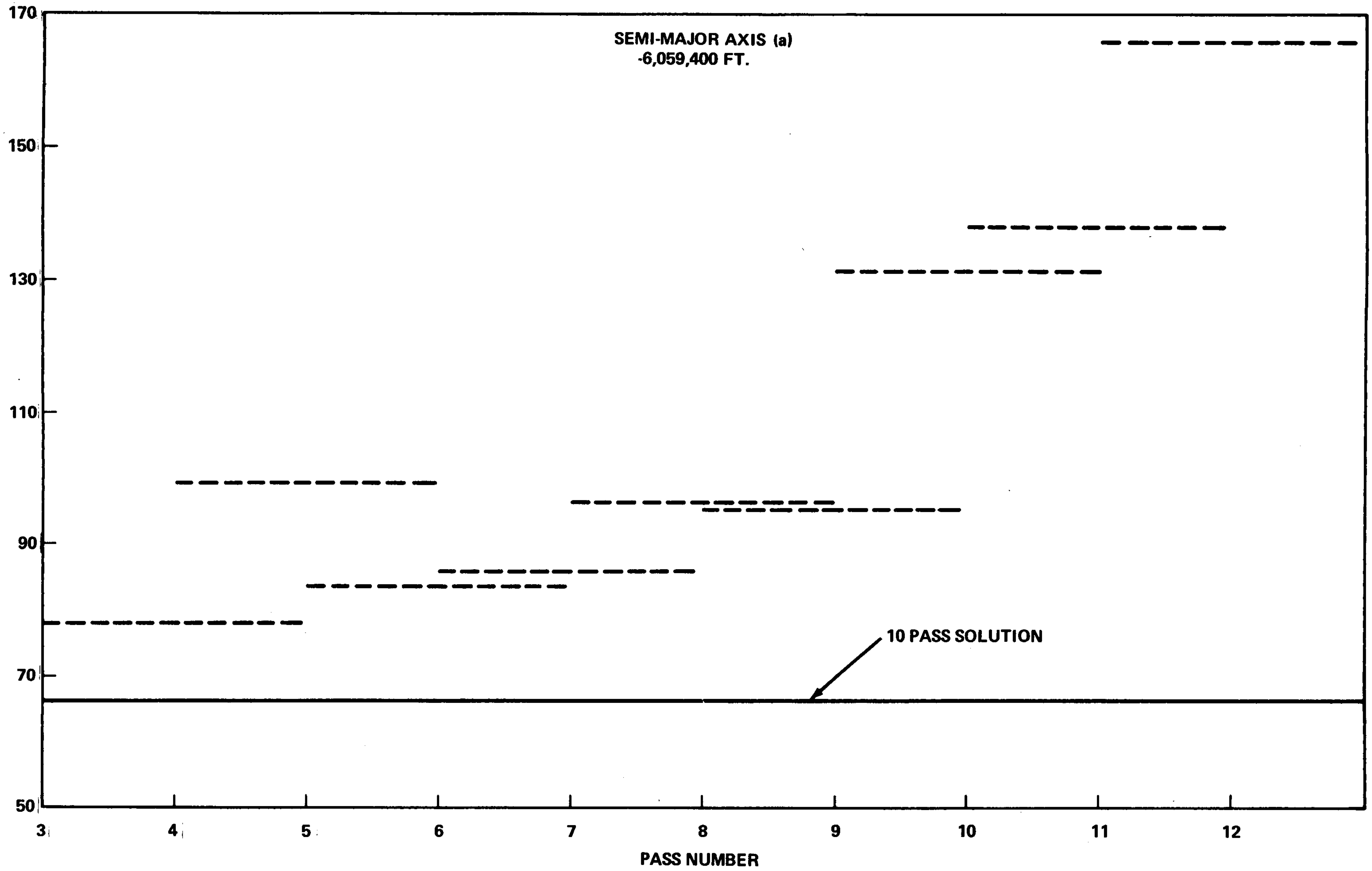

SEMI-MAJOR AXIS (a)
-6,059,400 FT.
10 PASS SOLUTION
170
150
130
110
90
70
50
3
4
5
6
7
8
9
10
11
12
PASS NUMBER

FIGURE 4A

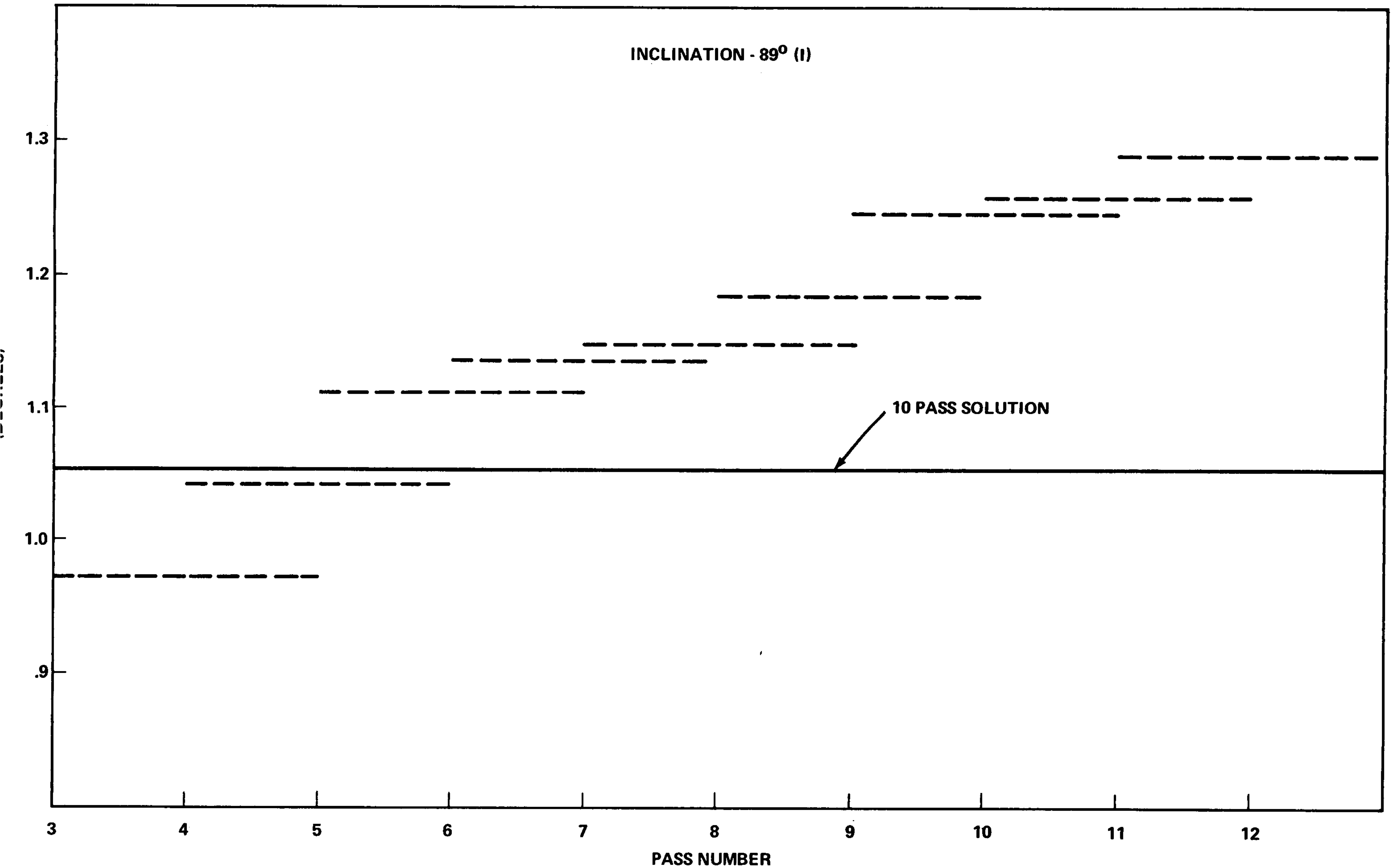

INCLINATION - 89° (I)
1.3
1.2
1.1
1.0
.9
10 PASS SOLUTION
3
4
5
6
7
8
9
10
11
12
PASS NUMBER

FIGURE 4B

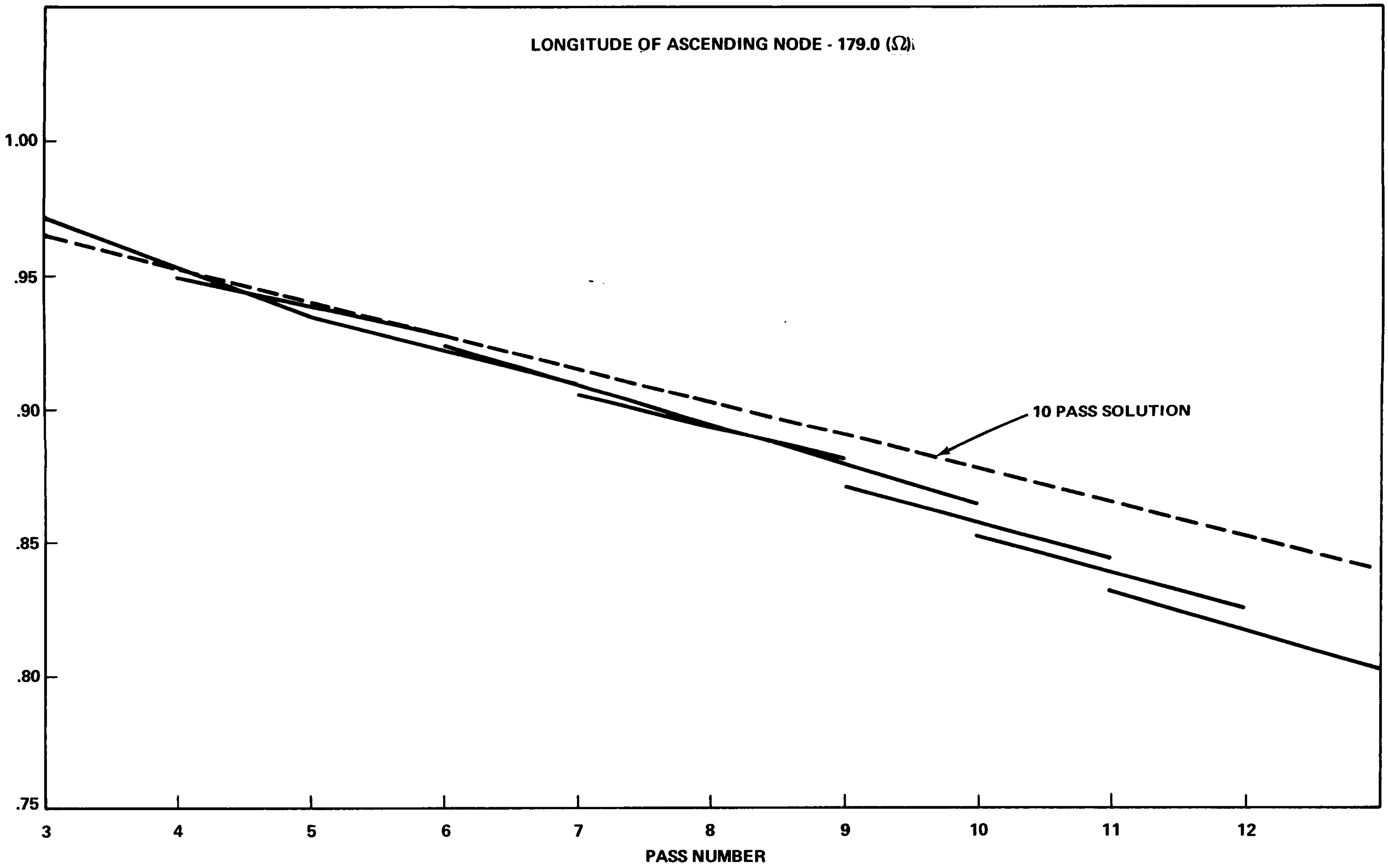
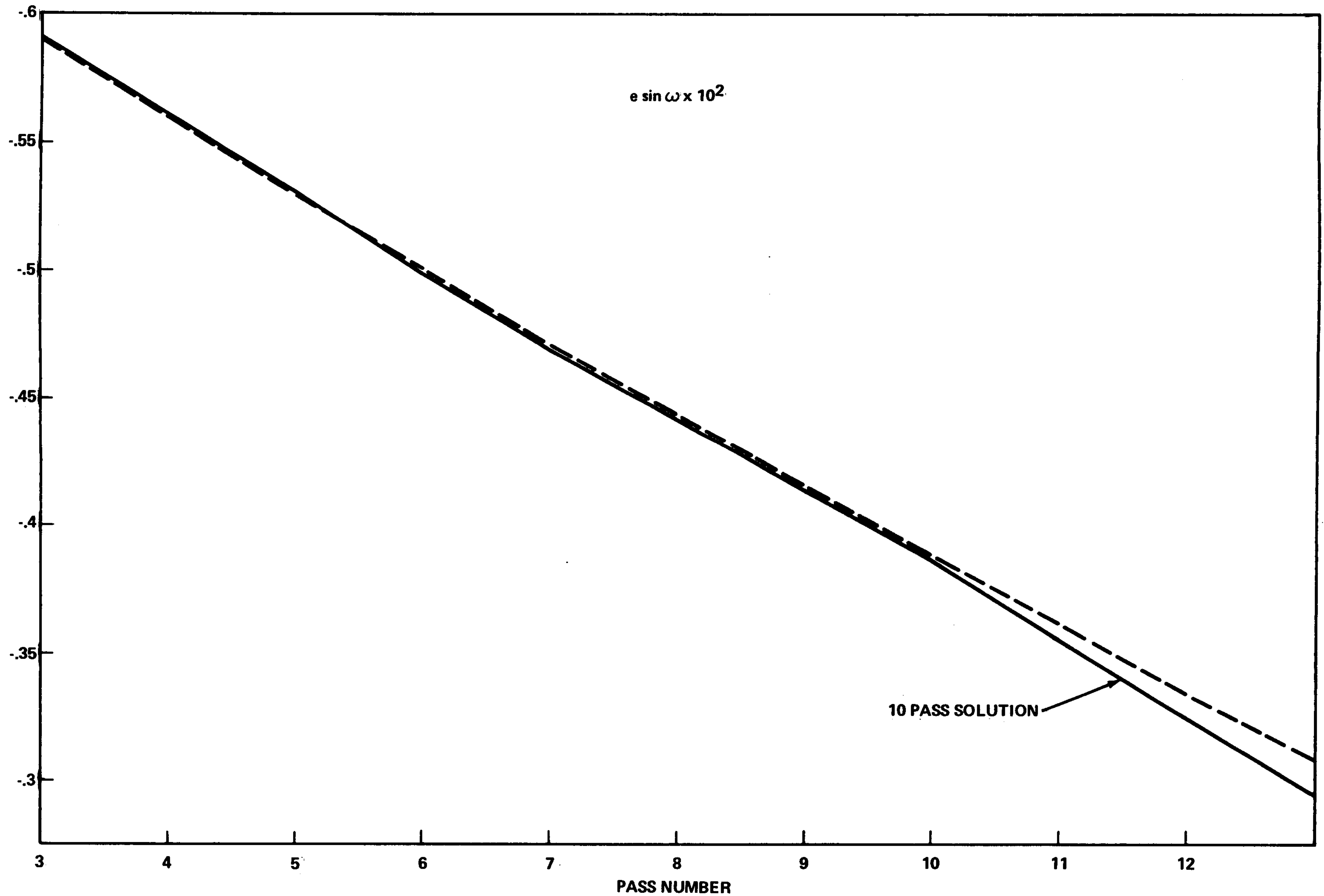

LONGITUDE OF ASCENDING NODE - 179.0 (Ω)
10 PASS SOLUTION
1.00
.95
.90
.85
.80
.75
3
4
5
6
7
8
9
10
11
12
PASS NUMBER

FIGURE 4C



-.6
-.55
-.5
-.45
-.4
-.35
-.3
e sin ω x 10²
10 PASS SOLUTION
3
4
5
6
7
8
9
10
11
12
PASS NUMBER

FIGURE 4D

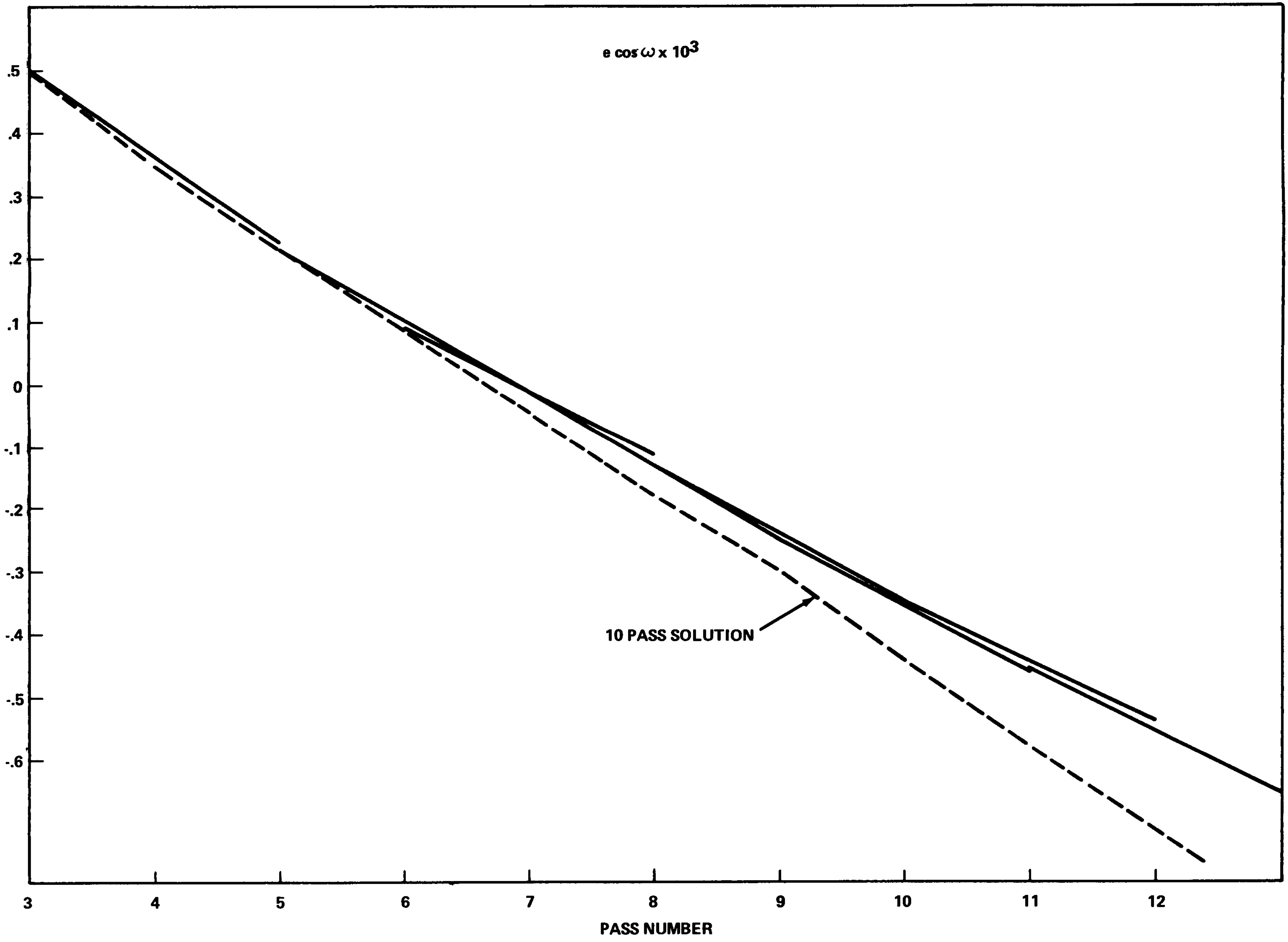

e cos ω x 10^3
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
-.6
10 PASS SOLUTION
3
4
5
6
7
8
9
10
11
12
PASS NUMBER

FIGURE 4E

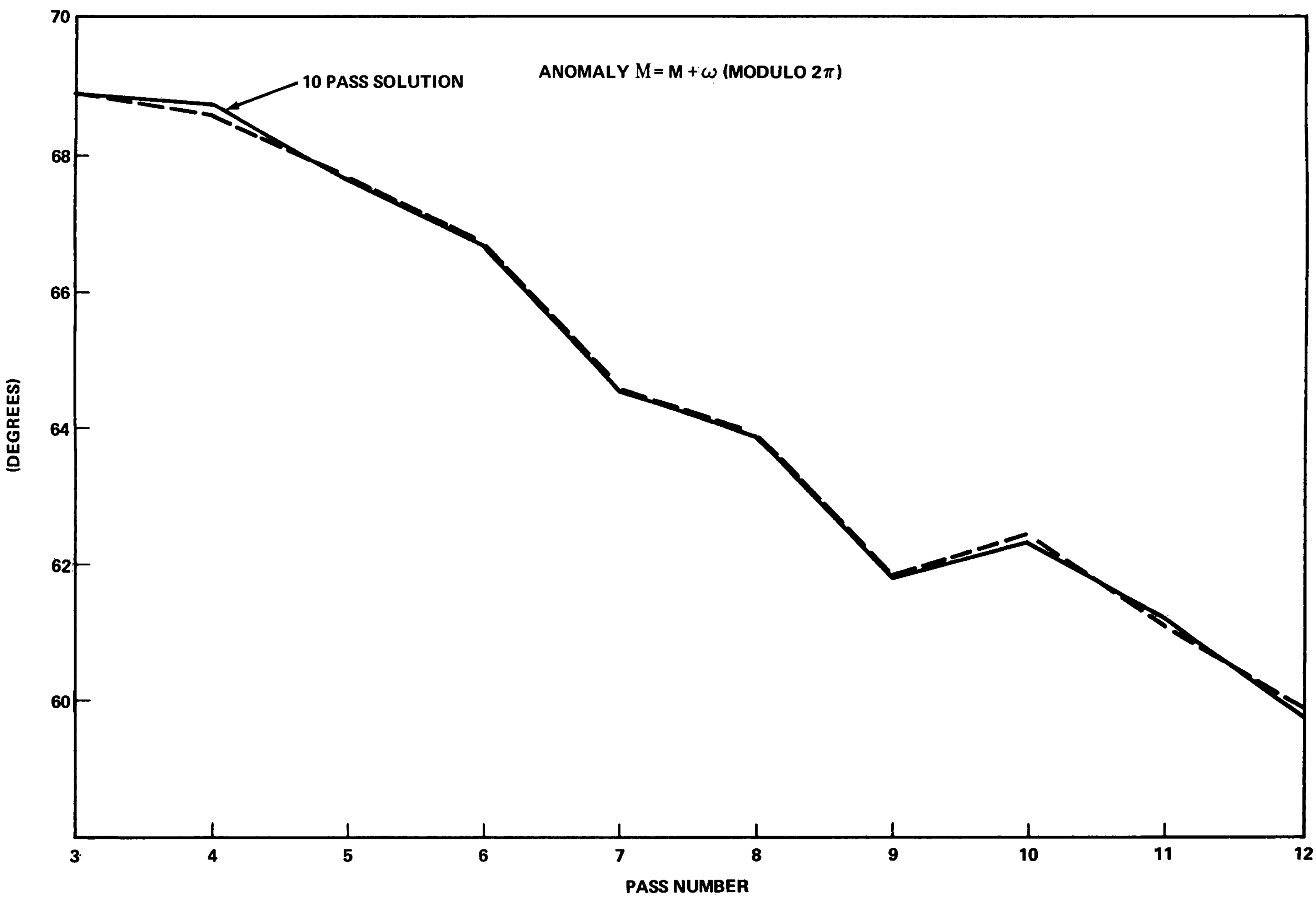

70
68
66
64
62
60
(DEGREES)
10 PASS SOLUTION
ANOMALY M = M + ω (MODULO 2π)
3
4
5
6
7
8
9
10
11
12
PASS NUMBER

FIGURE 4F